Name:___________________________

SM3 2.2: Fundamental Theorem of Algebra

__Problems:__ Find the complete linear factorization for each polynomial.

1) $x^2 + 16$

$$(x+4i)(x-4i)$$

2) $135x^3 + 40$

$$5(27x^3+8)$$

$$5(3x+2)(9x^2-6x+4)$$

$$45(3x+2)\left(x-\frac{1}{3}+\frac{i\sqrt{3}}{3}\right)\left(x-\frac{1}{3}-\frac{i\sqrt{3}}{3}\right)$$

3) $x^2 + 2x - 63$

$$(x+9)(x-7)$$

4) $8x^3 - 1$

$$(2x-1)(4x^2+2x+1)$$

$$4(2x-1)\left(x+\frac{1}{4}+\frac{i\sqrt{3}}{4}\right)\left(x+\frac{1}{4}-\frac{i\sqrt{3}}{4}\right)$$

5) $2x^2 - 7x - 15$

$$(2x+3)(x-5)$$

6) $125x^3 + 64$

$$(5x+4)(25x^2-20x+16)$$

$$25(5x+4)\left(x-\frac{2}{5}+\frac{2i\sqrt{3}}{5}\right)\left(x-\frac{2}{5}-\frac{2i\sqrt{3}}{5}\right)$$

7) $4x^3 + 12x^2 + 9x$

$$x(2x+3)^2$$

8) $7x^3 + 21x^2 - 6x - 18$

$$(7x^2-6)(x+3)$$

$$(\sqrt{7}x+\sqrt{6})(\sqrt{7}x-\sqrt{6})(x+3)$$

9) $12x^2 + 26x + 12$

$$2(2x+3)(3x+2)$$

10) $30x^3 - 6x^2 - 30x + 6$

$$6(5x^3-x^2-5x+1)$$

$$6(x^2-1)(5x-1)$$

$$6(x+1)(x-1)(5x-1)$$

11) $x^3 + 81x$

$x(x + 9i)(x - 9i)$

12) $18x^3 + 42x^2 + 3x + 7$

$(6x^2 + 1)(3x + 7)$

$(\sqrt{6}x + i)(\sqrt{6}x - i)(3x + 7)$

13) $x^4 + 5x^2 + 6$

$(x^2 + 3)(x^2 + 2)$

$(x + i\sqrt{3})(x - i\sqrt{3})(x + i\sqrt{2})(x - i\sqrt{2})$

14) $x^6 - 64$

$(x^3 - 8)(x^3 + 8)$

$(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$

$(x - 2)(x + 1 - i\sqrt{3})(x + 1 + i\sqrt{3})$
$(x + 2)(x - 1 - i\sqrt{3})(x - 1 + i\sqrt{3})$

15) $x^4 - 13x^2 + 36$

$(x^2 - 4)(x^2 - 9)$

$(x + 2)(x - 2)(x + 3)(x - 3)$

16) $36x^{44} - x^{43} - 21x^{42}$

$x^{42}(36x^2 - x - 21)$

$x^{42}(4x + 3)(9x - 7)$

Find the complete linear factorization for each polynomial with the given factor(s).

17) $x^3 + 4x^2 + x - 6; (x + 2)$

$(x + 2)(x^2 + 2x - 3)$

$(x + 2)(x - 1)(x + 3)$

18) $x^3 - 7x^2 + 2x + 40; (x - 5)$

$(x - 5)(x^2 - 2x - 8)$

$(x - 5)(x - 4)(x + 2)$

19) $6x^3 - 29x^2 + 23x + 30; (x - 3)$

$(x - 3)(6x^2 - 11x - 10)$

$(x - 3)(3x + 2)(2x - 5)$

20) $15x^3 - 37x^2 - 86x - 24; (3x + 1)$

$(3x + 1)(5x^2 - 14x - 24)$

$(3x + 1)(5x + 6)(x - 4)$

21) $2x^3 + 7x^2 + 32x + 112; (x + 4i)$

$(x^2 + 16)(2x + 7)$

$(x + 4i)(x - 4i)(2x + 7)$

22) $3x^3 - 20x^2 + 42x - 20; (3x - 2)$

$(3x - 2)(x^2 - 6x + 10)$

$(3x - 2)(x - 3 - i)(x - 3 + i)$

23) $3x^3 - 6x^2 - 255x - 462; (x - 11)$

$$3(x^3 - 2x^2 - 85 - 154)$$

$$3(x - 11)(x^2 + 9x + 14)$$

$$3(x - 11)(x + 2)(x + 7)$$

24) $10x^3 + 25x^2 + 40x + 100; (x - 2i)$

$$5(2x^3 + 5x^2 + 8x + 20)$$

$$5(x^2 + 4)(2x + 5)$$

$$5(x + 2i)(x - 2i)(2x + 5)$$

25) $x^4 + 27x^2 - 324; (x + 3)(x - 3)$

$$(x + 3)(x - 3)(x^2 + 36)$$

$$(x + 3)(x - 3)(x + 6i)(x - 6i)$$

26) $x^4 + 10x^3 + 29x^2 + 50x + 120; \left(x - i\sqrt{5}\right)$

$$(x^2 + 5)(x^2 + 10x + 24)$$

$$\left(x + i\sqrt{5}\right)\left(x - i\sqrt{5}\right)(x + 4)(x + 6)$$

27) $6x^4 - 31x^3 - 94x^2 + 89x + 210; (3x - 5)(x - 7)$

$$(x - 7)(6x^3 + 11x^2 - 17x - 30)$$

$$(x - 7)(3x - 5)(2x^2 + 7x + 6)$$

$$(x - 7)(3x - 5)(2x + 3)(x + 2)$$

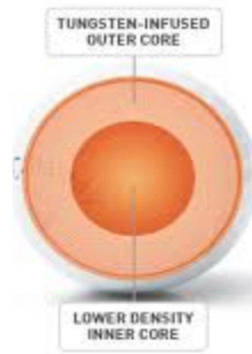
28) $6x^4 - 11x^3 - 44x^2 + 77x + 14; (x - 2)(6x + 1)$

$$(x - 2)(6x^3 + x^2 - 42x - 7)$$

$$(x - 2)(6x + 1)(x^2 - 7)$$

$$(x - 2)(6x + 1)\left(x + \sqrt{7}\right)\left(x - \sqrt{7}\right)$$

29) Austin's golf ball design company, *Hit Our Balls and Yell "Fore!"*, has finished a new style of ball. The inner core of the ball is a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ that varies in radius to accommodate players with different types of swings. This core is suspended within the outer casing and then the ball is filled with tungsten-infused material that makes up the outer core. The ball, itself, has a radius of 1 inch.



a) Determine the volume of the entire ball.

$$V = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi\ in^3$$

b) Build polynomial $v_{oc}(r)$ to describe the volume of the outer core in terms of the radius of the inner core.

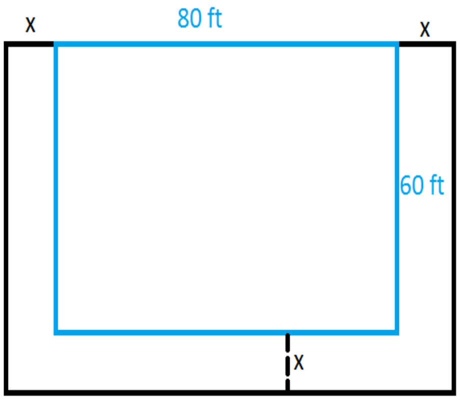
$$v_{oc}(r) = V - V_{ic} = \frac{4}{3}\pi - \frac{4}{3}\pi r^3$$

c) Factor $v_{oc}(r)$ completely.

$$\frac{4}{3}\pi(1 - r^3)$$

$$\frac{4}{3}\pi(1 - r)(1 + r + r^2)$$

$$\frac{4}{3}\pi(1 - r)\left(r + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(r + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$



30) After graduating from high school, Jamison goes on to a lucrative career in engineering, as inspired by his math teacher. To say "thanks", he decides to build his math teacher a rectangular wave pool on 7488 ft^2 of space that was previously used for something unimportant, like student parking. This pool will have a length of 60ft and a width of 80ft. The size of the walkway around the pool is uniform but unknown. The walkway does not completely surround the pool; you do not want swimmers hopping in where the waves are generated!

a. Construct polynomial $A(x)$ that describes the surface area being made into the pool with the walkway.

$$A(x) = lw$$

$$(x + 60)(2x + 80)$$

$$2x^2 + 200x + 4800$$

b. Find the value of x.

$2x^2 + 200x + 4800 = 7488$	Set equal to the given area
$2x^2 + 200x - 2688 = 0$	Use subtraction so that our equation equals zero
$x^2 + 100x - 1344 = 0$	Divide by 2
$(x - 12)(x + 112) = 0$	Factor

$x = 12, -112$; We use the solution, $x = 12ft$, because we are not interested in building a walkway with negative length.